Bohr Model of H-atom

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Bohr Model of Hydrogen Atom :-

There are three basic postulates of this model :-

(1) Every atom consist of a central core called nucleus, in which entire mass of the atom are concentrated. A suitable number of electrons revolve around the nucleus in circular orbits.

The central force required for revolution is provided by the electrostatic force of attraction between the electron and the nucleus.

centrifugal force = Electrostatic force of attraction

$$\boxed{\frac{mv^2}{r} = \frac{kze}{r^2}}$$

$$\begin{cases} k = \frac{1}{4\pi\varepsilon_0} \\ z = 1 \end{cases}$$

(2) According to Bohr, electron can revolve only in certain discrete non radiating orbits, called stationary orbits, for which total angular momentum of the revolving electron is an integral multiple of $\frac{h}{2\pi}$, where h is plank's constant.

Thus the angular momentum of a orbitting electron is quantised.

$$\boxed{mvr = \frac{nh}{2\pi}} \qquad n = 1, 2, 3, \ldots$$

Here n is called Principle quantum number.

(3) The emission/absorption of energy occurs only when an electron jumps from one of its specified non-rotating orbit to another. The difference in the total energy of electron in the two orbits is absorbed when the electron jumps from an inner to an outer orbit and emitted when electron jumps from outer to inner orbit.

$$h\nu = E_2 - E_1$$

Where ν is the frequency of radiation emitted on jumping from outer to inner orbit of energy E_2 and E_1 respectively.

Radius of Bohr's stationary orbits :-

We know ; for stationary orbits :-

$$mvr = n\hbar = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$$

Bohr's first Postulates :

$$\frac{mv^2}{r} = \frac{k(ze)(e)}{r^2}$$

$$\frac{m}{\cancel{r}} \cdot \frac{n^2h^2}{4\pi^2 m^2 r^{\cancel{2}}} = \frac{kze^2}{\cancel{r^2}}$$

$$\boxed{r = \frac{n^2h^2}{4\pi^2 mke^2}}$$

For H-atom ; $z = 1$ → no. of e^- = no. of p

It shows that - $r \propto n^2$

Hence, the radius of stationary orbits are in the ratio $1^2 : 2^2 : 3^2$ and so on

i.e. $1:4:9:$------ . Clearly the stationary orbits are not equally spaced.

Velocity of electron in Bohr's stationary orbit :-

As we know that ;

$$\frac{mv^2}{\cancel{r}} = \frac{kZe^2}{r^{\cancel{2}}}$$

$$v^2 = \frac{kze^2}{mr}$$

We know ;

$$r = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

$$v^2 = \frac{kze^2}{\cancel{m}\left(\frac{n^2h^2}{4\pi^2 \cancel{m} kze^2}\right)}$$

$$v^2 = \frac{(kze^2)(4\pi^2 kze^2)}{n^2h^2}$$

$$v^2 = \frac{4\pi^2 Z^2 k^2 e^4}{n^2 h^2}$$

$$\boxed{v = \frac{2\pi Z k e^2}{nh}}$$

For H-atom ; $Z = 1$

$$\boxed{v = \frac{2\pi k e^2}{nh}}$$

$k = \frac{1}{4\pi\varepsilon_0}$

As $v \propto \frac{1}{n}$, hence the orbital velocity of electron in outer orbits is smaller as compared to its value in the inner orbits.

Frequency of electron in Bohr's stationary orbits (ν) :-

It is the number of revolutions completed per second by the electron in a stationary orbit, around the nucleus. It represented by ν.

$$v = r\omega = r(2\pi\nu)$$

$$\nu = \frac{v}{2\pi r} = \frac{2\pi k Z e^2}{nh \cdot 2\pi r}$$

$v = \frac{2\pi k Z e^2}{nh}$

$$\boxed{\nu = \frac{k Z e^2}{nhr}}$$

$\longrightarrow$ For H-atom $Z = 1$

$$\boxed{\nu = \frac{k e^2}{nhr}}$$

The frequency of electron in subsequent stationary orbits is smaller as $\nu \propto \frac{1}{n}$

Total energy of electron in Bohr's stationary orbit :-

$\longrightarrow$ classical model. (Non-rel. model)

↳ kinetic energy of electron :-

$$K.E. = \frac{1}{2} m v^2$$

$\left[\text{By } \frac{mv^2}{r} = \frac{kZe^2}{r^2} \right]$

$$K.E. = \frac{1}{2}mv^2$$

$$K.E. = \frac{kze^2}{2r}$$

$\left[\text{By } \frac{mv^2}{r} = \frac{kze^2}{r^2} \right]$

↳ Potential energy of electron :-

$$P.E. = \frac{k(ze)(-e)}{r} = \frac{-kze^2}{r} = -2(K.E.)$$

↳ Total Energy :-

$$E = K.E. + P.E.$$

$$E = \frac{kze^2}{2r} - \frac{kze^2}{r} = \frac{-kze^2}{2r} \quad \text{——— (1)}$$

As we know ; $r = \dfrac{n^2h^2}{4\pi^2 kze^2 m}$

Substituting value of 'r' in eq^n (1)

$$E = \frac{-2\pi^2 m k^2 z^2 e^4}{n^2h^2} \rightarrow 13.6 \text{ eV}$$

By substituting the standard values, we get ; (For H-atom ; $z = 1$)

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$$\boxed{E = \frac{-13.6 \text{ eV}}{n^2}} \qquad E \propto \frac{1}{n^2}$$

Hence the total energy of electron in a stationary orbit is negative, which means that the electron is bound to the nucleus and is not free to leave it.

↳ When $n = 1$, then this state of lowest energy of atom is called ground state.

The energy of this state is $E_1 = -13.6$ eV.

↳ Therefore, the minimum energy required to free the electron from the ground state of hydrogen atom is 13.6 eV. This is called ionisation energy of hydrogen atom.

As n increases, the value of negative energy decreases. i.e. energy is progressively larger in the outer orbits.